

IMPACT FORCE IDENTIFICATION USING FREQUENCY RESPONSE FUNCTION

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ABSTRACT

This paper presents an approach using Frequency Response Function to inversely determine impact forces in structural component and machine members. A Frequency Response Function (FRF) is a transfer function expressed in the frequency domain obtained using either measured data or analytical functions. A frequency response function was generated for all input and output combinations for a cantilever beam using force data and acceleration data in the frequency domain. The force data was acquired using impact hammer and their responses were measured using accelerometer in LabVIEW software. Further the impact peak force of the hammer was compared with the force recreated using FRF at those locations and their results were plotted and compared. Further a prototype vehicle model was developed and an attempt was made to recreate the forces using generated FRF for single wheel, 2 wheels and 4 wheels using impact hammer and results were plotted and compared. This particular method of force identification was later applied for obtaining the maximum impact force using FRF when the vehicle model was dropped from a particular height.

KEYWORDS: Frequency Response Function, Fast Fourier Transform, Time domain, Frequency Domain & LabVIEW

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1. INTRODUCTION

Any structural member or a machine component is designed based on estimated loads rather than actual loads. The design will be safe only if the capacity of the member or component to withstand the loads is higher than the estimated loads or the failure takes place. So lots of trial and error method is to be carried out for proper and successful design. This problem can be overcome by measuring the responses first and then determining the loads using these measured responses which is called Inverse problem. These loads correspond to actual loads on the basis of which a member or a component is designed. Frequency response function approach involves measurement of spectrum of responses during operation and estimation of excitation forces corresponding to these measured responses.

Frequency Response Functions are generally used to describe the input-output relationships of any system. This function describes the input-output relationship between two points on a structure as a function of frequency. It is considered to be a tool for performing vibration analysis and testing. It is a measure of how much displacement,

velocity or acceleration response a structure has at an output degree of freedom, per unit excitation force at an input degree of freedom. A Frequency Response Function is defined as the ratio of Fourier Transform of an output response $A(\omega)$ divided by the Fourier transform of the input force $F(\omega)$ that caused the output as shown in Figure 1.

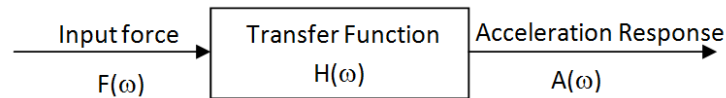


Figure 1: Frequency Response Function Model

The relationship in Figure 1 can be represented in the form of Equation 1

$$A(\omega) = H(\omega) \times F(\omega) \quad (1)$$

$F(\omega)$ is the input force as a function of the angular frequency ω . $H(\omega)$ is the transfer function. $A(\omega)$ is the acceleration response function. Each function is a complex function, which can be also represented in terms of magnitude and phase.

2. EXPERIMENTAL SET UP FOR FRF GENERATION ON A CANTILEVER BEAM

The experimental set consisted of a rectangular sectioned aluminium strip having a gauge length 400mm. The specimen was fixed to a table at one end in such a way that the strip resembled a cantilever beam. 3 different locations were marked on the beam at distances of 120mm from the fixed end. Two sets of readings were taken from a particular input, output combination of different locations identified on the cantilever beam. First set was to obtain the Frequency Response Function and the second set was to recreate the forces obtained using Frequency Response Function. The cantilever beam was impacted using impact hammer and the response was measured using accelerometer as shown in Figure 2.

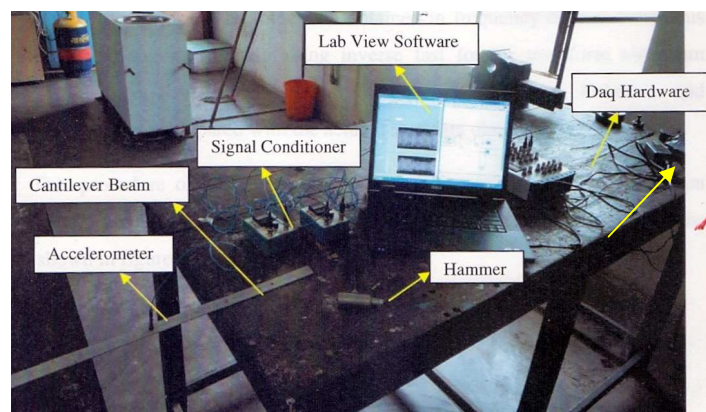


Figure 2: Experimental Set up of a Cantilever Beam for FRF Generation and Force Recreation

The force and acceleration data obtained in time domain were transformed into frequency domain using Fast Fourier Transform in MatLAB software and Frequency Response Function was obtained for that particular input output combination in MatLAB software. One more set of force and acceleration readings were taken at the same location of the cantilever beam where only acceleration data was transformed from time domain to frequency domain. Using the frequency response function identified for that particular input output combination, force data was obtained in frequency domain using acceleration data. The force data in frequency domain was converted to time domain force data

using Inverse Fast Fourier Transform (IFFT) in MatLAB software and the obtained force data using FRF was compared with the force data obtained using impact hammer.

A series of experiments were conducted for all possible input output combinations at different locations of the cantilever beam. One such possible input output combination is as shown in Figure 3.

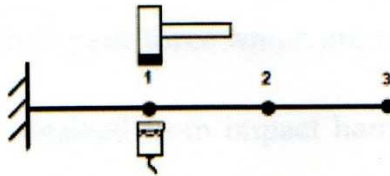


Figure 3: Input and Output both at Location 1

Here both the force measurement and accelerometer response is obtained at location 1 of the cantilever beam. The force and acceleration data were collected for 2 seconds. These data were converted from time domain to the frequency domain using FFT in MatLAB software and Frequency Response Function was calculated as shown in Table 1.

Table 1: FRF for Input and Output both at Location 1(H_{11})

Time(s)	Force(N)	Acceleration (m/s ²)	Force in Frequency Domain F (ω)	Acceleration in Frequency Domain A (ω)	Frequency Response Function H (ω)
0.88368	0.508731	0.138514	1.879-1.075i	0.423-0.111i	54.93-8.708i
0.88416	0.652722	0.135264	0.9763-0.1266i	0.1217-0.522i	32.13+8.804i
0.88464	0.488967	0.129339	2.5243-2.384i	-0.060+0.360i	155.36+112.32i
0.88464	0.392973	0.142018	-2.945-1.244i	0.3074-0.531i	260.99-109.45i

Table 2 shows the actual force data obtained from impact hammer and recreated forces obtained using FRF for both input output combination at location 1.

Table 2: Force Data Recreated using Calculated FRF for H_{11}

Time(s)	Actual Force from Hammer(N)	Acceleration (m/s ²)	Acceleration in Frequency Domain A(ω)	Frequency Response Function H(ω)	Force Recreated using FRF in Frequency Domain F(ω)	Force Recreated using FRF in Time Domain (N)
0.88368	0.594614	2.386146	107.03+1.632i	54.93-8.708i	1.896+0.33i	2.472780
0.88416	8.26285	38.405852	-66.53-35.02i	32.13+8.804i	-2.203-0.486i	8.943382
0.88464	12.29178	-148.67544	86.62+54.75i	155.36+112.32i	0.533-0.033i	13.127847
0.88512	10.68247	-181.48097	-30.21-79.04i	260.99-109.45i	0.009-0.298i	12.942948

From Table 2, it is clear that the peak force from hammer was 12.29178N and peak force recreated using FRF was 13.127847N at 0.88464 seconds.

Figure 4 shows the plot of actual force data obtained using impact hammer and force data recreated using FRF.

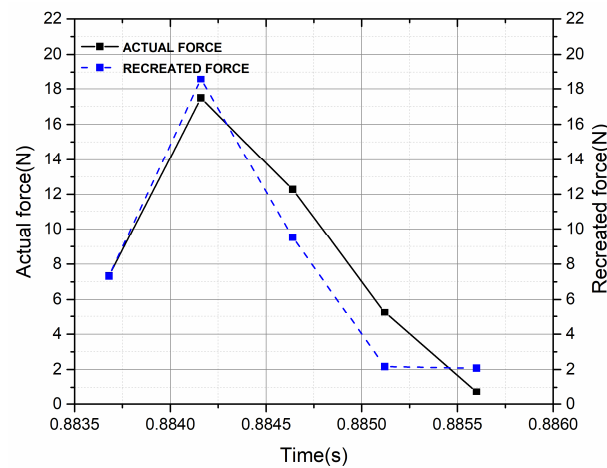


Figure 4: Actual Force Data from Impact Hammer vs Recreated Force Data using FRF for H_{11}

From Figure 4, it can be observed that the actual force data obtained from impact hammer and the recreated force data using FRF very well agreed with each other with a maximum error in peak force of about 6%. The above procedure was repeated for input and output both at location 3 (H_{33}). The results were tabulated as shown in Table 3 and 4 and the graphs were plotted as shown in Figure 5.

Table 3: FRF for Input and Output both at Location 3 (H_{33})

Time (s)	Force (N)	Acceleration (m/s^2)	Force in Frequency Domain $F(\omega)$	Acceleration in Frequency Domain $A(\omega)$	Frequency Response Function $H(\omega)$
0.32736	0.695652	-9.441176	$75.67+41.73i$	$-112.85-47.47i$	$-1.408+0.149i$
0.32784	0.695652	-10.411765	$67.03-40.80i$	$-103.25+64.25i$	$-1.549+0.015i$
0.32832	0.608695	-10.313725	$16.23-87.29i$	$-10.88+121.12i$	$-1.363+0.128i$
0.3288	0.478260	-10.039216	$-75.07-58.12i$	$94.36+76.95i$	$-1.281-0.032i$

Table 4: Force Data Recreated using Calculated FRF for H_{33}

Time (s)	Actual Force from Hammer (N)	Acceleration (m/s^2)	Acceleration in Frequency Domain $A(\omega)$	Frequency Response Function $H(\omega)$	Force Recreated using FRF in Frequency Domain $F(\omega)$	Force Recreated using FRF in Time Domain (N)
0.32736	14.608695	2.088235	$-29.50-16.14i$	$-1.408+0.149i$	$19.51+13.53i$	14.955932
0.32784	17.913043	-0.823529	$-27.54+18.83i$	$-1.549+0.015i$	$17.89-11.97i$	17.364665
0.32832	17.130434	-8.303922	$2.503+30.57i$	$-1.363+0.128i$	$0.281-22.39i$	17.015687
0.3288	16.826087	-5.519608	$30.45+14.48i$	$-1.281-0.032i$	$-24.02-10.69i$	15.308145

From Table 4, it is clear that the peak force from hammer was 17.913043N and peak force recreated using FRF was 17.364665N at 0.32784 seconds. Figure 6 shows the plot of actual force data obtained using impact hammer and force data recreated using FRF.

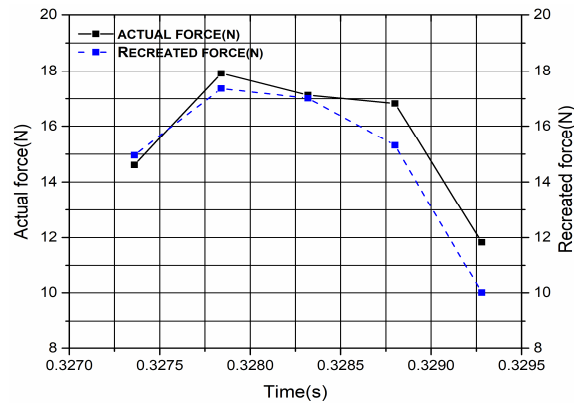


Figure 5: Actual Force Data from Impact Hammer vs Recreated Force Data using FRF for H_{33}

From Figure 5, it can be observed that the actual force data obtained from impact hammer and the recreated force data using FRF very well agreed with each other with a maximum error in peak force of about 3%.

3. FRF GENERATION ON A PROTOTYPE VEHICLE MODEL

Chassis and axles in a vehicle are designed to take static and dynamic loads acting on the vehicle. Usually in a vehicle, wheels experiences impact force and axles which are connected to the wheels experience more vibration because of the impact. Frequency Response Function plays a very important role for predicting forces even in complex structures like a vehicle model for identifying and recreating forces. Hence an effort has been made to recreate forces acting on a prototype vehicle model made of solid axles and steel wheels along with the bearing.

3.1 Experimental Set Up

Figure 6 shows an experimental set up to measure force and acceleration data on a vehicle model. The vehicle model was suspended in Universal Testing Machine using a guitar string having high stiffness along with mild steel square plate being welded to the wheels at the bottom. Care was taken so that the vehicle model was steady without being subjected to tilt. The accelerometer was fixed at the centre of the vehicle model on axles and the centre of the plate was impacted using hammer. The load was assumed to be equally distributed to all the 4 wheels to which the plate was welded.

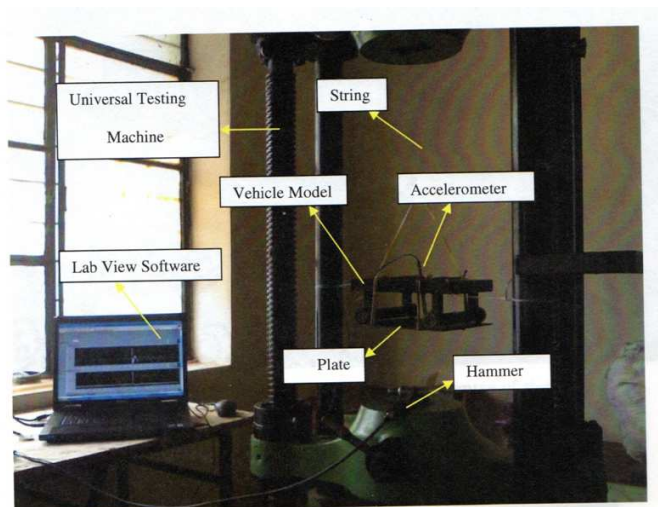


Figure 6: Experimental Set up for 4 Wheel Impact with Plate Attached

The force and acceleration data were converted from time domain to frequency domain to obtain Frequency Response Function and using this Frequency Response Function the force data were recreated and compared with the actual peak force obtained using impact hammer. Table 5 shows the FRF using force and acceleration data for 4 wheel impact and results were plotted as shown in Figure 7.

Table 5: FRF of the Vehicle Model for 4 Wheel Impact

Time(s)	Force(N)	Acceleration (m/s ²)	Force in Frequency Domain $F(\omega)$	Acceleration in Frequency Domain $A(\omega)$	Frequency Response Function $H(\omega)$
1.09104	0.3478	0.169477	5.998+4.524i	1.063+2.349i	0.301+0.1644i
1.09152	0.246159	-0.232853	5.363-4.434i	-1.151-2.392i	0.091-0.370i
1.092	0.305449	0.350987	-1.397+5.318i	0.185+2.711i	0.468-0.157i
1.09248	0.35627	0.133672	0.317-6.828i	-0.332-2.829i	0.411-0.067i

Table 6: Force Data Recreated using Calculated FRF for 4 Wheel Impact

Time(s)	Actual Force from Hammer(N)	Acceleration (m/s ²)	Acceleration in Frequency Domain $A(\omega)$	Frequency Response Function $H(\omega)$	Force Recreated Using FRF In Frequency Domain $F(\omega)$	Force recreated using FRF in Time Domain(N)
1.09104	12.04497	2.328169	-0.060-0.143i	0.301+0.1644i	-0.354-0.283i	11.566
1.09152	13.0642	2.947432	-0.516+0.0058i	0.091-0.370i	-0.339-1.309i	13.054
1.092	13.46794	4.317202	-0.047-0.334i	0.468-0.157i	0.124-0.672i	13.566
1.09248	13.08114	5.641418	-0.357+0.088i	0.411-0.067i	-0.879+0.069i	13.075

From Table 6, it is clear that the peak force from hammer was 13.46794N and peak force recreated using FRF was 13.566 N at 1.092 seconds. Figure 7 shows the plot of actual force data obtained using impact hammer and force data recreated using FRF for 4 wheel impact.

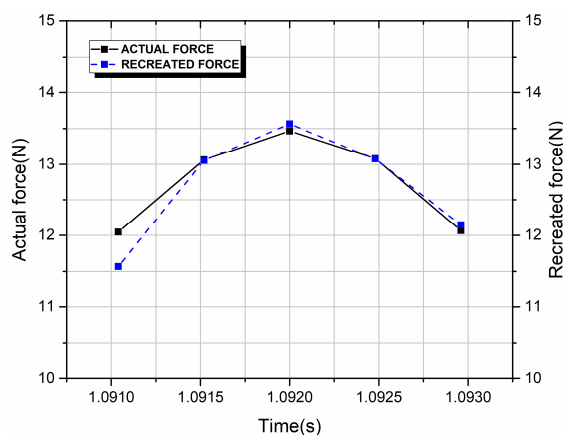


Figure 7: Actual Force Data from Impact Hammer vs Recreated Force Data using FRF for 4 Wheel Impact

From Figure 7, it can be observed that the actual force data obtained from impact hammer and the recreated force data using FRF very well agreed with each other with a maximum error in peak force of about 1%.

After recreating the force data obtained using FRF for 4 wheel impact, experiment was conducted on measurement of impact force of the vehicle model when dropped from a certain height. Initially the accelerometer was fixed at the centre of the vehicle model on axles and the centre of the plate was impacted using hammer for obtaining FRF

similar to that of 4 wheel impact. Later the vehicle model was dropped from a certain height. Care was taken such that the vehicle model was not subjected to any tilt. Only accelerometer data was recorded and using acceleration data and FRF obtained, the impact force of the vehicle model when dropped was obtained as shown in Table 7 and 8.

Table 7: FRF of Vehicle Model during 4 Wheel Impact

Time (s)	Force (N)	Acceleration (m/s^2)	Force in Frequency Domain F (ω)	Acceleration in Frequency Domain A (ω)	Frequency Response Function H (ω)
0.4368	0.409913	0.144821	10.02-14.44i	-1.960+1.288i	-0.123-0.049i
0.43728	0.463557	0.14361	1.181-0.497i	1.571-0.508i	1.283+0.1095i
0.43776	0.271569	0.140297	10.03-12.69i	-2.193+0.327i	-0.099-0.0938i

Table 8: Force Data Obtained using FRF during Vehicle Model Dropping

Time (s)	Acceleration (m/s^2)	Acceleration in Frequency Domain A (ω)	Frequency Response Function H (ω)	Force Calculated using FRF in Frequency Domain F (ω)	Force obtained using FRF in Time Domain(N)
0.4368	-171.572825	330.82-240.92i	-0.123-0.049i	-1624.02+2600i	41.19
0.43728	64.846029	-184.36-334.86i	1.283+0.1095i	-164.66-246.76i	60.38
0.43776	117.178856	-329.32+126.92i	-0.099-0.0938i	1117.94-2318.6i	32.52

From Table 8, it is clear that the maximum impact force obtained during dropping of the vehicle model was found to be 60.38N at 0.43728 seconds.

4. RESULTS AND DISCUSSIONS

An effort has been made in order to recreate the forces for different input output combinations of cantilever beam at all possible locations as well as on vehicle model employing frequency response function approach. The maximum impact force was also found during dropping of the vehicle model using FRF approach. Table 9 shows the maximum error in actual peak forces obtained using impact hammer and recreated peak forces obtained using FRF for different input output combinations of a cantilever beam. Table 10 shows the maximum error in actual and recreated peak forces for single wheel, 2 wheels and 4 wheel impact.

Table 9: Actual and Recreated Peak Forces for different Input Output Combinations on a Cantilever Beam

Frequency Response Function (H_{ij})	Time (s)	Peak Actual Force from Impact Hammer (N)	Peak Recreated Force from FRF (N)	% error
H_{11}	0.88464	12.29178	13.127847	6.3
H_{12}	0.76456	10.88844	11.575056	5.9
H_{13}	0.70368	8.422072	8.935368	5.7
H_{31}	0.200641	19.565217	19.113721	2.3
H_{32}	0.45408	16.34783	15.897587	2.75
H_{33}	0.32784	17.913043	17.364665	3

From Table 9, it can be observed that the magnitude of recreated forces and actual forces very well agreed with each other. The magnitude of the actual forces and recreated forces were found to be maximum at the location where the acceleration was maximum i. e free end of the cantilever beam at location 3 and occurred. More acceleration means good FRF result. The percentage error was also found to be minimum about 3% when the accelerometer was placed at location 3 near to the free end rather than location 1 which was near to the fixed end. Thus, it can be concluded that FRF depends on

the position of force applied and response measured.

Table 10: Actual and Recreated Peak Forces for Single, 2 Wheels and 4 Wheel Impact

Wheel Combination Subjected to Impact	Peak Time (s)	Peak Actual Force (N)	Peak Recreated Force (N)	% error
Single Wheel Impact	1.0848	19.16829	19.383426	1.1
2 Wheels Impact	0.78384	26.49773	26.177693	1.2
4 Wheels Impact	1.092	13.46794	13.566795	0.7

From Table 10 it can be observed that the actual peak force from the hammer and recreated peak force using FRF for different wheel combinations very well agreed with each other with maximum %error of 1%. However the error observed was due to the reason that instead of directly hitting the wheels, the impact force was measured by hitting at the centre of the strip attached to 2 wheels in case of 2 wheel impact and hitting at the centre of the plate welded to all the 4 wheels in case of 4 wheel impact.

5. CONCLUSIONS

In the present work a methodology has been presented to measure the forces inversely on a structural member and prototype vehicle model from the measured responses and frequency response function data. The maximum impact force experienced when the vehicle model was dropped was also measured using frequency response function data and acceleration data.

From the experimental results on a cantilever beam, it was observed that the percentage error in peak force recreated when the accelerometer being placed near to the free end i. e. location 3 was lesser about 3% when compared to that placed near to the fixed end i. e. location 1 about 5%. This is due to the reason that free end experiences more acceleration when compared to fixed end.

From the experimental results on the vehicle model, for single, 2 wheel impact and 4 wheel impact, it was observed that the peak force recreated using FRF and actual peak force from impact hammer agreed very well with a maximum error of about 1%.

This approach of impact force measurement using FRF can be extended to measure operating loads on machine members and to detect damage in any structure. This method can be used in determining the unknown forces in real time applications for a real vehicle model.

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